PVC Creep Data Fitting for NOvA

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OUTLINE

- Purpose
  - Why are we doing this?
- Collecting Data
  - How are we doing this?
- Analyzing Data
  - What does this data tell us?
- Conclusion
- What implications does this have?

PURPOSE

- What is creep?
  - Creep is the deformation of material due to stress over time that can lead to mechanical failure
- Why do we care about creep?
  - The PVC that will be used in the NO*ν*A detector will be subject to stress caused by the weight of the detector for the duration of the detector's run. Therefore, it will undergo creep deformation.
  - We need to know whether or not this deformation will be enough to cause mechanical failure of the PVC that makes up the structure of the detector

COLLECTING DATA

- The samples are much wider where they are clamped to the stand so that the creep will essentially only take place in the long center part
- There is a lever arm at the bottoms of the samples on which to put weights for causing stress
- Data was taken for 9 different stresses, with 2 PVC samples at each stress, for a total of 18 samples
- Data was taken periodically over 188 days

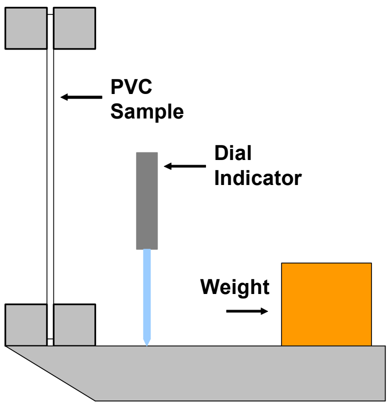


Figure 1: Diagram of creep test stand

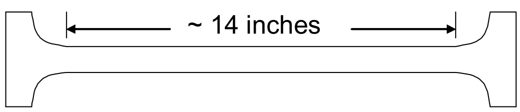


Figure 2: Example of PVC sample

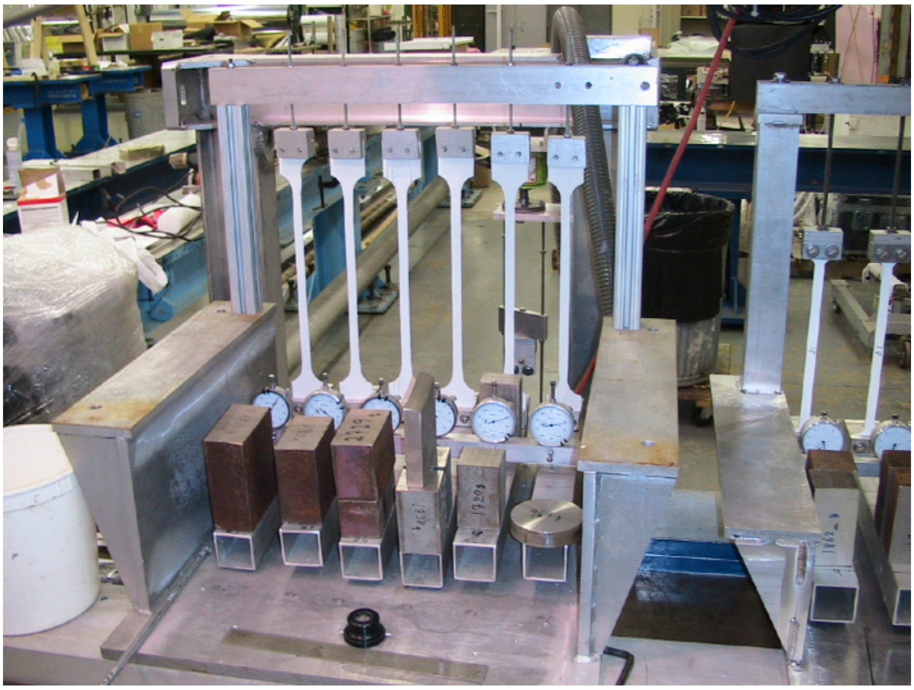


Figure 3: Picture of a test stand in use

INITIAL DATA

- This graph represents the stress-strain curves of the PVC at select times. Each curve represents the strain at every stress at the time indicated in the legend.
- Strain $= \frac{\Delta L}{L}$

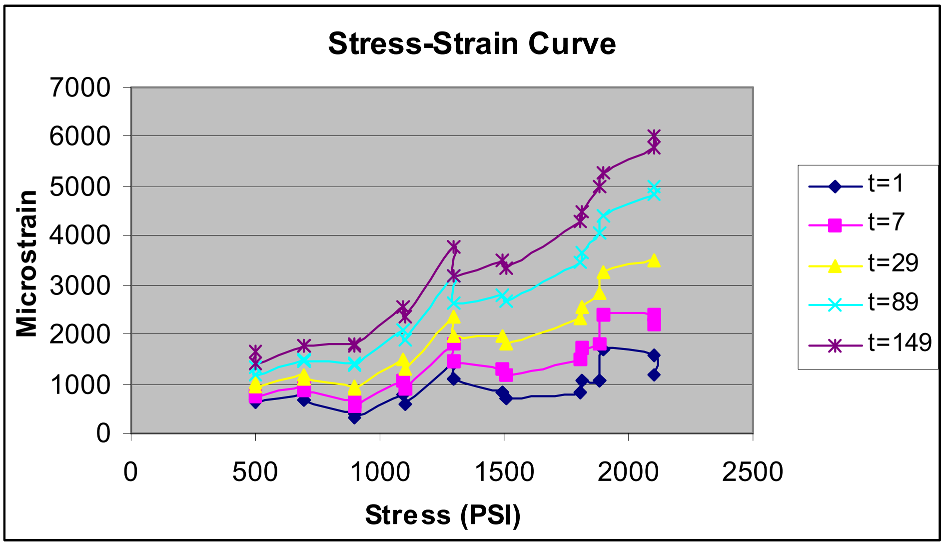


Figure 4: Stress-strain curves at various times

CREEP

- Creep is the deformation of material due to stress over time that can lead to mechanical failure
- There are three stages of creep:
  - Primary: strain rate decreases
  - Secondary: strain rate is approximately constant (linear region)
  - Tertiary: strain rate increases (preceeds creep rupture)
- Since our data does not represent the tertiary or secondary creep region, the data should resemble the primary creep region

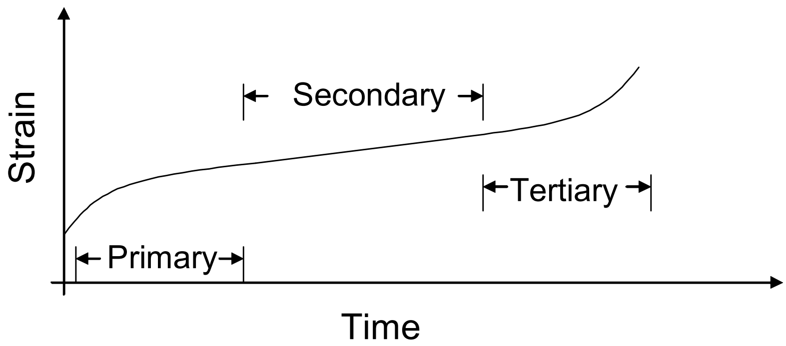


Figure 5: Sketch of a general strain curve with creep regions indicated

CREEP MODULUS

- What is it?
  - The creep modulus is the ratio of applied stress to strain due to creep

$$\text{Creep Modulus} = \frac{Stress}{Strain}$$

- What do we want to see?
  - Since we want the minimum amount of strain for a given stress, we would like to see the highest creep modulus possible

LINEAR VS. NON-LINEAR FITS

- With a linear fit, stress is proportional to strain
- The fact that creep is split into three different stages suggests that the process is not linear with respect to time. Since it doesn't make sense, I did not try a linear fit with respect to time
- A linear fit refers to being linear with respect to stress
- The best fit for the stress-strain curve looks like it might be a straight line (although the data is quite rough so it is not clear), which suggests that a linear fit with respect to stress might be a good fit.
- I will fit the data with a linear fit with respect to stress, which will be shown later, however most of the fits shown will not be linear

GRAPH SCALE COMPARISON

- Figure 6 shows that the data does, in fact, resemble the primary creep stage
- This comparison should give you an idea of how these curves translate to a log scale graph
- The rest of the graphs in this presentation will be on a log scale unless otherwise noted

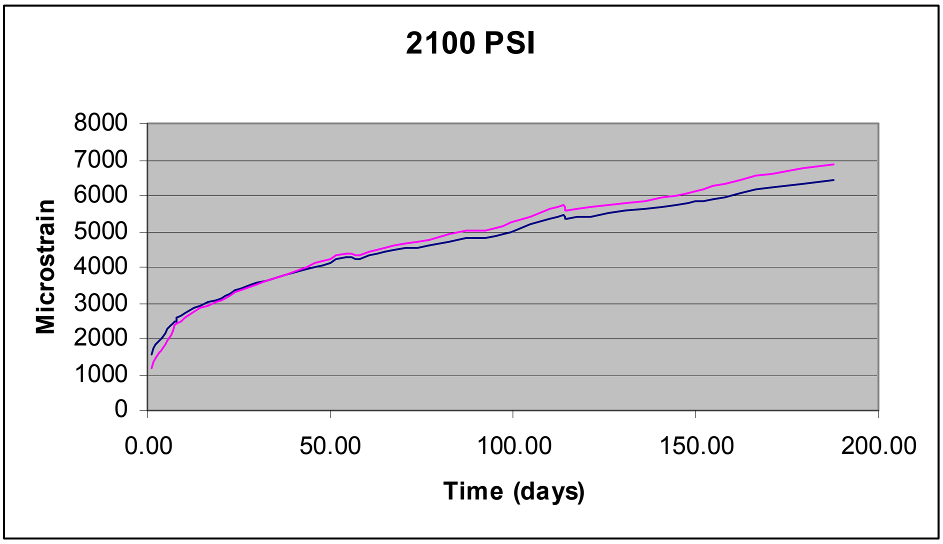


Figure 6: Strain curves for samples at 2100 PSI on a linear scale graph

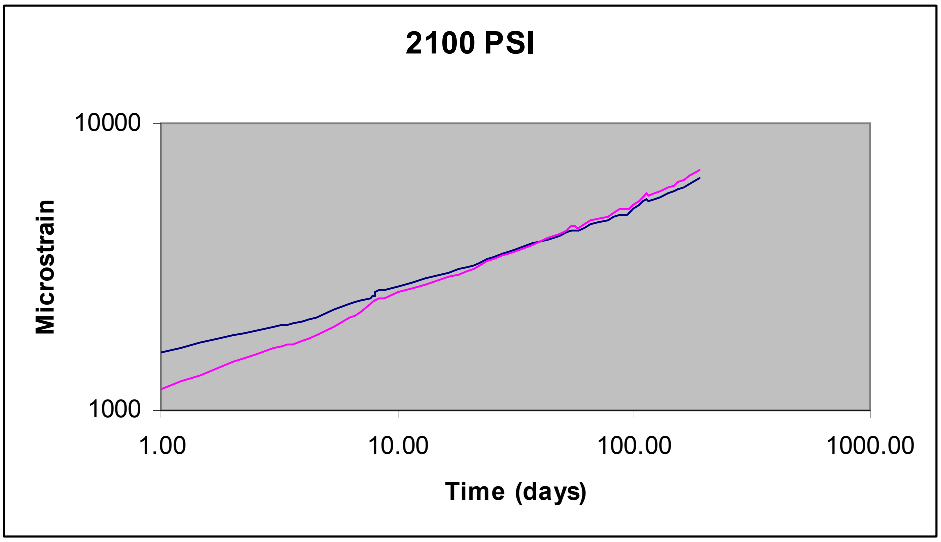


Figure 7: Strain curves for samples at 2100 PSI on a log scale graph

STRESS EFFECT

- It is important to notice that as the stress increases, the strain curve changes shape as well as magnitude
- At low stresses the strain curve is curved, and well represented by an exponential
- At high stresses the strain curve is a straight line, and well represented by a power law

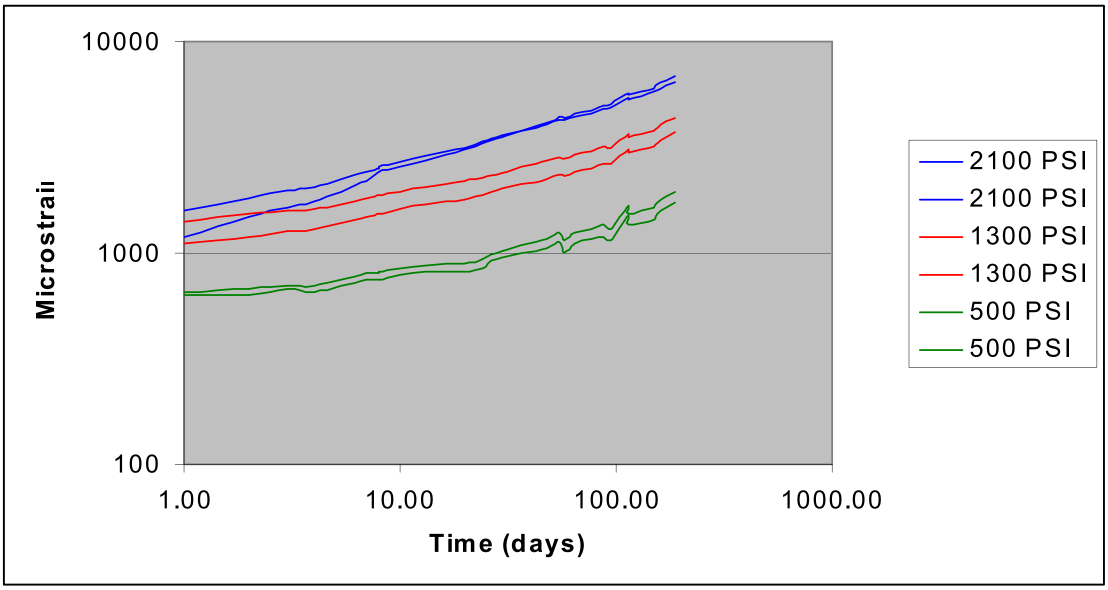


Figure 8: Strain curves at various stresses

MATHEMATICALLY MODELING CREEP

- Uniaxial creep is generally modeled by the separable equation

$$\varepsilon = f_1(\sigma) f_2(t) f_3(T)$$

- For this analysis, the temperature function will be considered to be an arbitrary constant. The temperature dependence is usually based on the Arrhenius law

$$f_3(T) = A \exp\left(\frac{-\Delta H}{RT}\right)$$

  where ΔH is the activation energy, R is the Boltzmann constant, and T is the absolute temperature. Because of how this formula applies the temperature correction, it makes sense to use it for samples that were each kept at a constant temperature (i.e. temperature change between samples). It does not make sense to use it for temperature change between measurements. Since the temperature of the samples was 22.5 deg C +/- 2 deg C, we decided that this was a small enough ΔT between measurements to use the raw data without applying a temperature correction.
- There are now test stations ready to start taking temperature dependent data in a controlled environment.

LINEAR VISCOELASTIC MODEL

- In this model, the first spring is used to represent the almost instantaneous deformation that occurs when a stress is applied to an object.
- The spring and dashpot, connected in parallel, represent the slower deformation that occurs due to a continued stress on an object over time.
- Since the result of the first spring is nearly instantaneous at t=0, it is easier to assume that it is instantaneous and analyze the spring-dashpot combination. Then you can let the effect of the first spring be represented by a constant

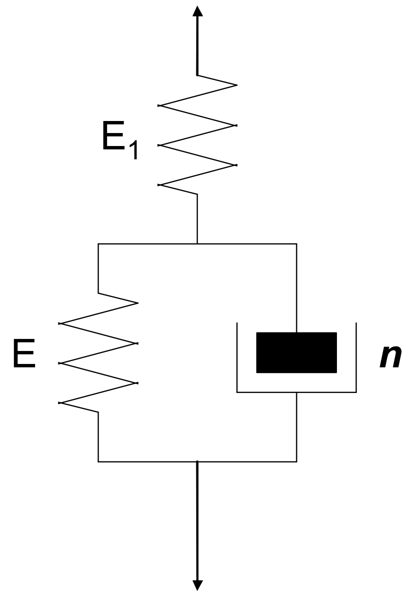


Figure 9: Diagram of the linear viscoelastic creep model

DESCRIBING THE MODEL

For a dashpot, velocity is proportional to force, so

$$\frac{dx}{dt} = \eta F$$

After doing a force balance, you're left with the differential equation

$$\frac{dx}{dt} = \eta(mg - Ex)$$

Which has the solution

$$x = C\exp(-\eta Et) + \frac{mg}{E}$$

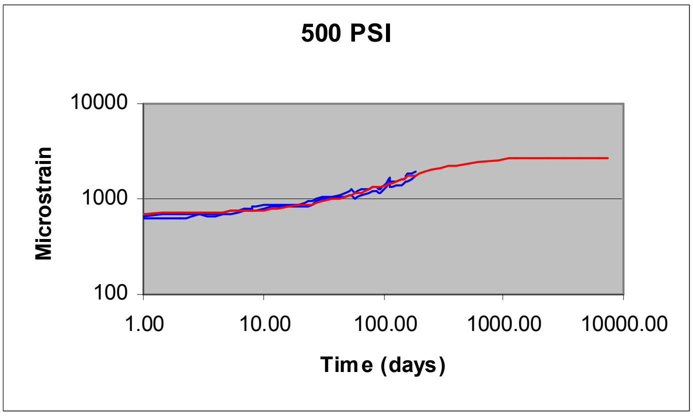
However, since all of the constants in the model are arbitrary, you can rewrite this more simply as

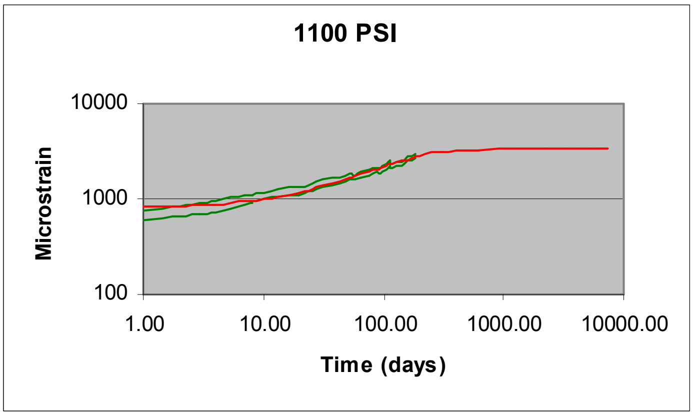
$$x = A + B\exp\left(-t/\tau\right)$$

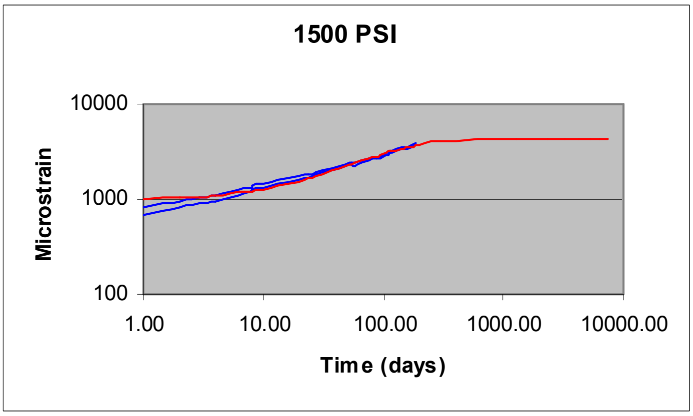
This model is implicitly linear with respect to stress, since spring compression is linear with respect to force, and dashpot velocity is linear with respect to force.

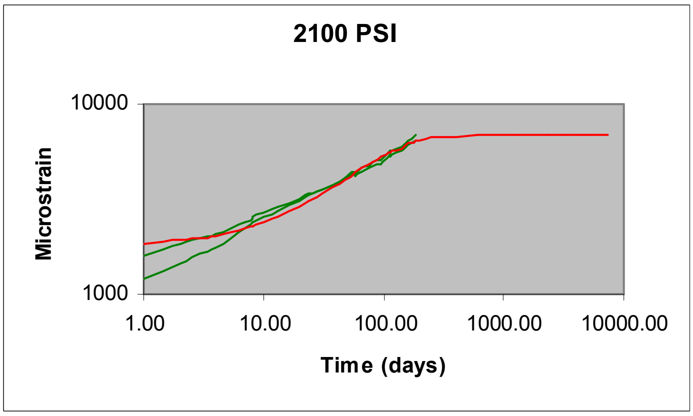
LINEAR VISCOELASTIC MODEL GRAPHS

For the rest of this document, fits are in red and all other colors represent data









Figures 10-13: Creep data fit using the linear viscoelastic model

GRAPH ANALYSIS

- The exponential seems to work well for low stress
- It is not explicitly a function of stress
  - As a result, each stress has to be fit individually
  - This causes the fit to lose predictive power, since you have can only use stresses that you've tested
- It does not fit well at high stress
- Needs a stress function to go with it, but what form?
  - Since the high stresses resemble a power law, and the low stresses resemble an exponential, those are the most logical functions to try
  - Based on the stress-strain curve, a linear relationship might be a good choice

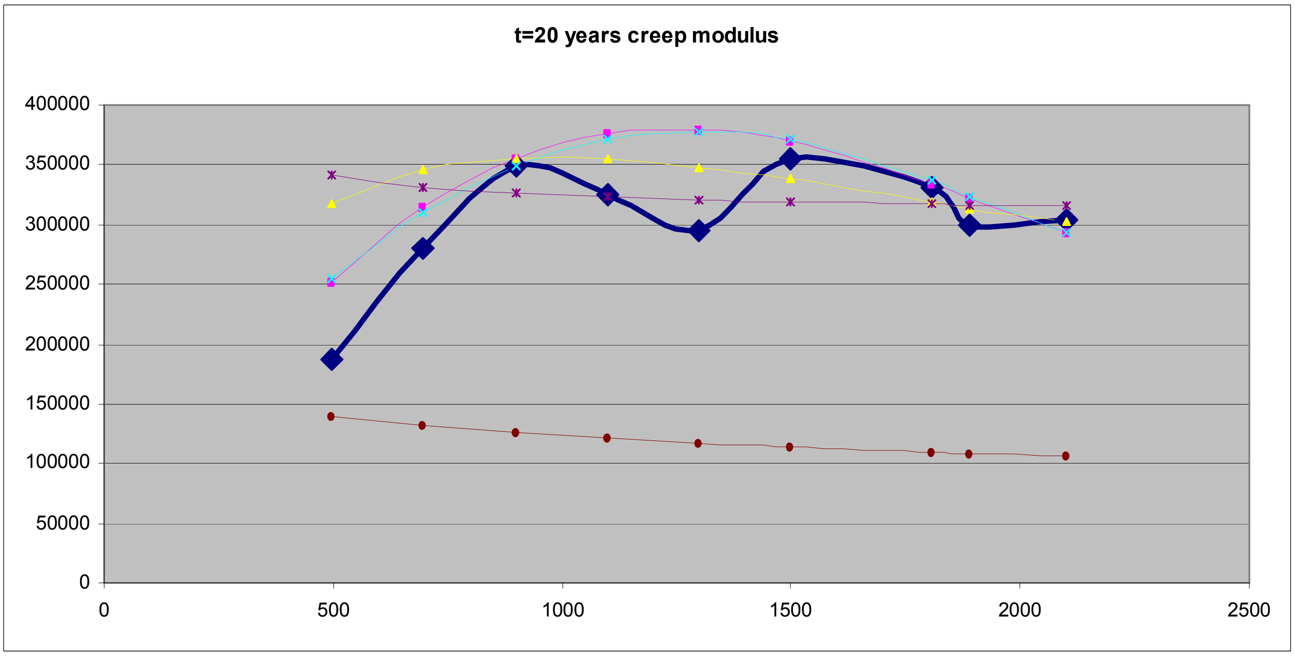
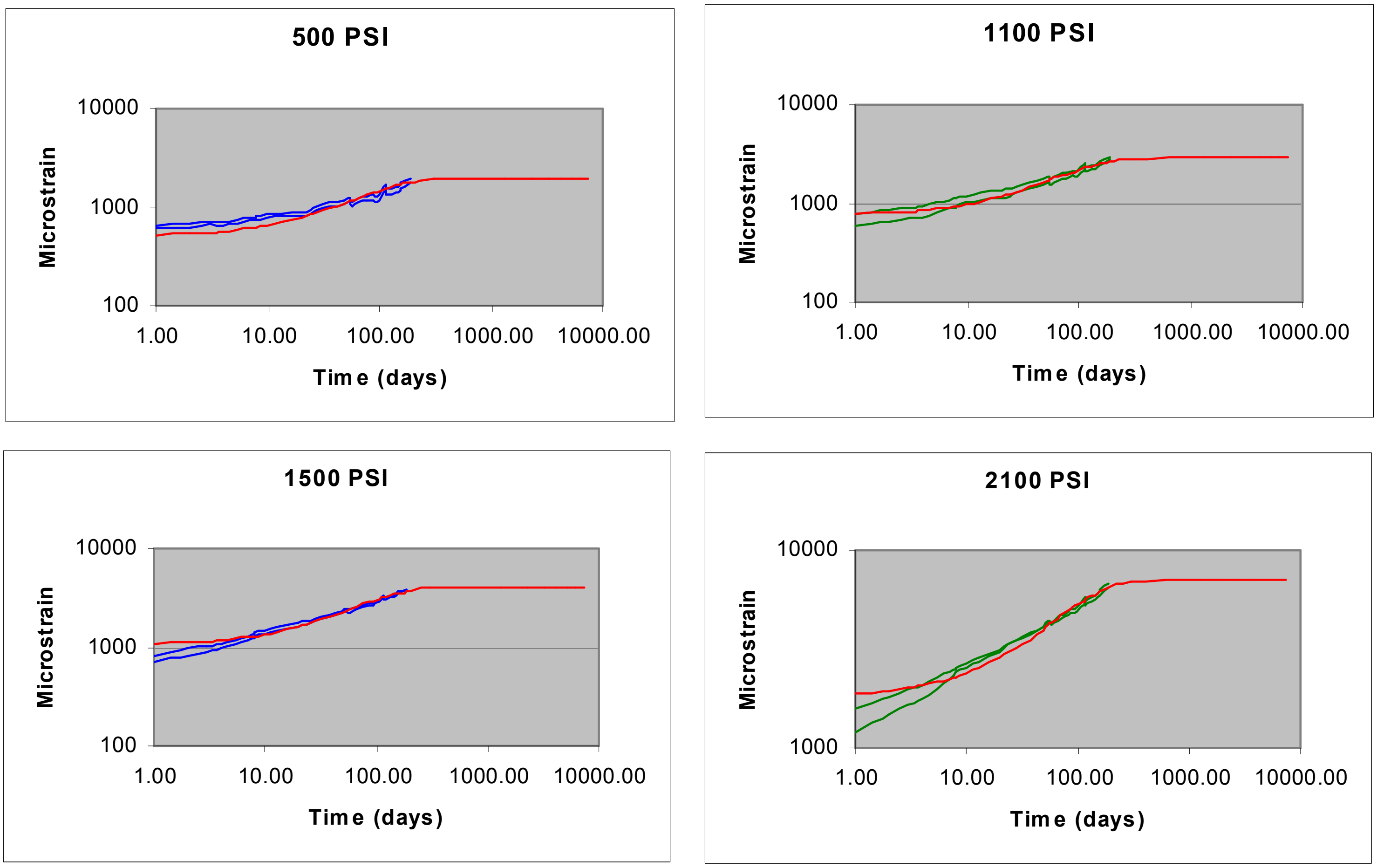


Figure 14: Stress vs. creep modulus at t = 20 years, predicted using linear viscoelastic model fit functions (bold blue line)

$$\varepsilon = \left(A + B\exp\left(-t/\tau\right)\right)\left(C + D\exp\left(\sigma/S\right)\right)$$



Figures 15-18: Creep data fit using an exponential time function and an exponential stress function

- This fit has the advantage of being explicitly stress dependent, unlike the linear viscoelastic model
- This fit has the same problems as the linear viscoelastic model, but it gives a worse fit

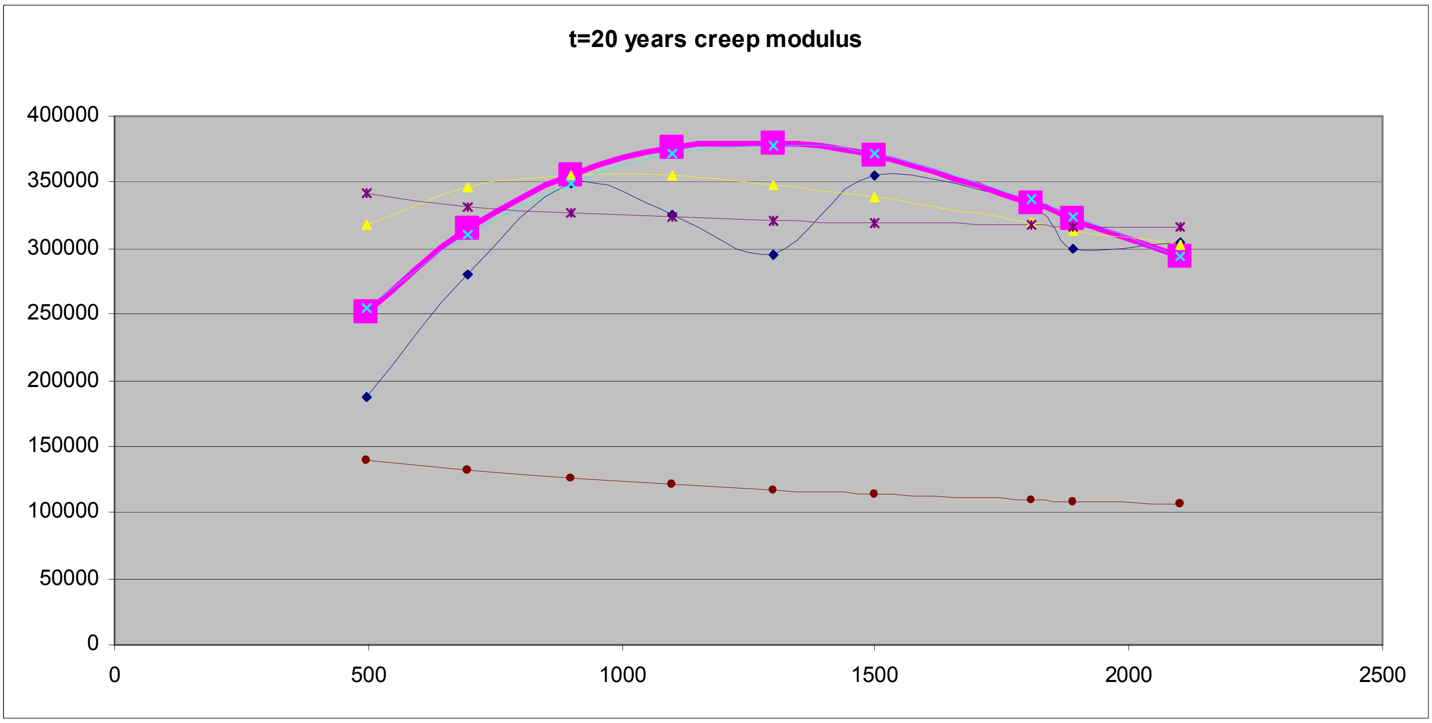
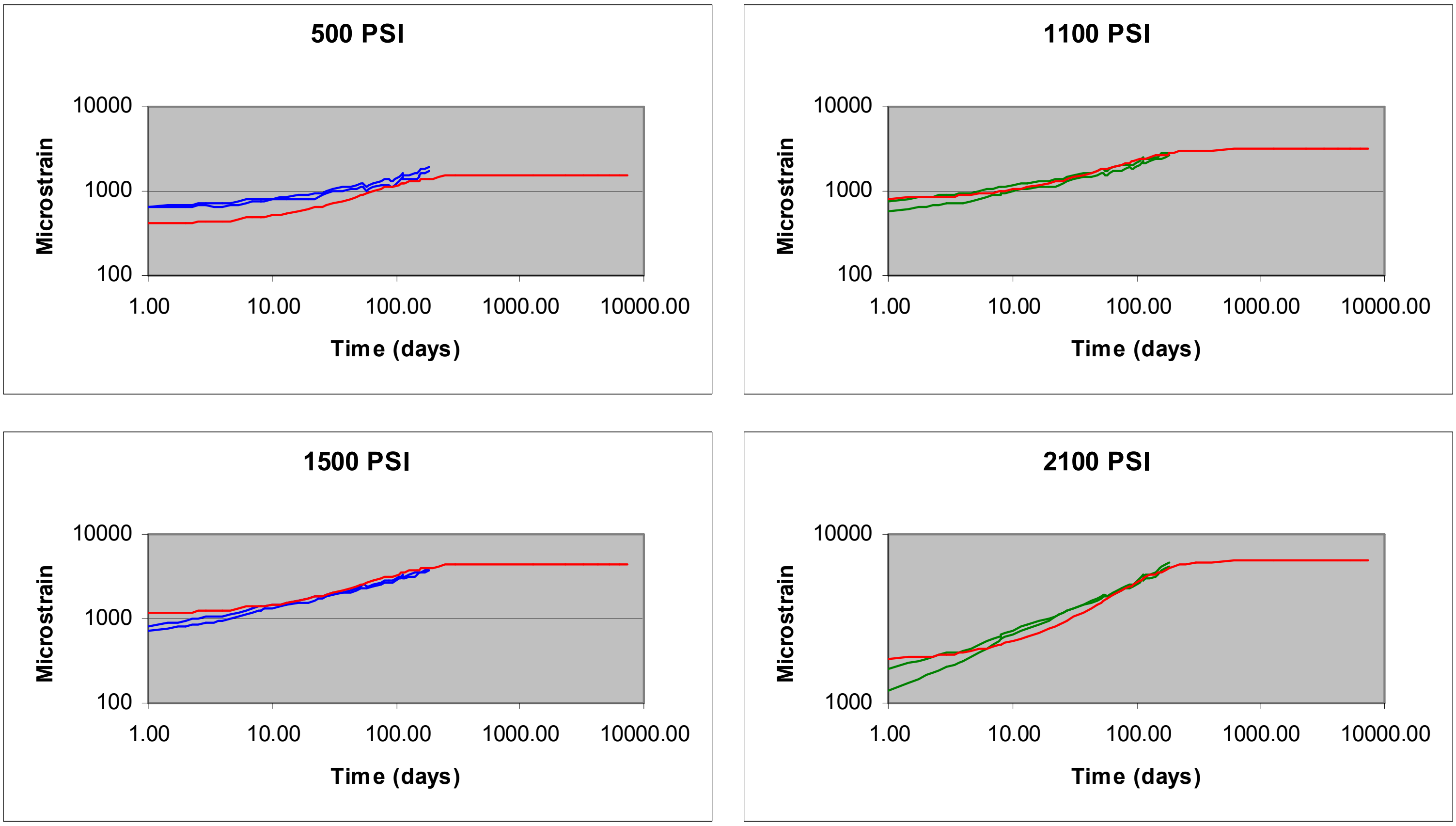


Figure 19: Stress vs. creep modulus at t = 20 years, predicted using an exponential stress and exponential time fit function (bold pink line)

$$\varepsilon = \left(\sigma + \sigma_0\right)^n \left(A + B \exp\left(-{}^{t}\!/_{\tau}\right)\right)$$



Figures 20-23: Creep data fit using an exponential time function and a power law stress function

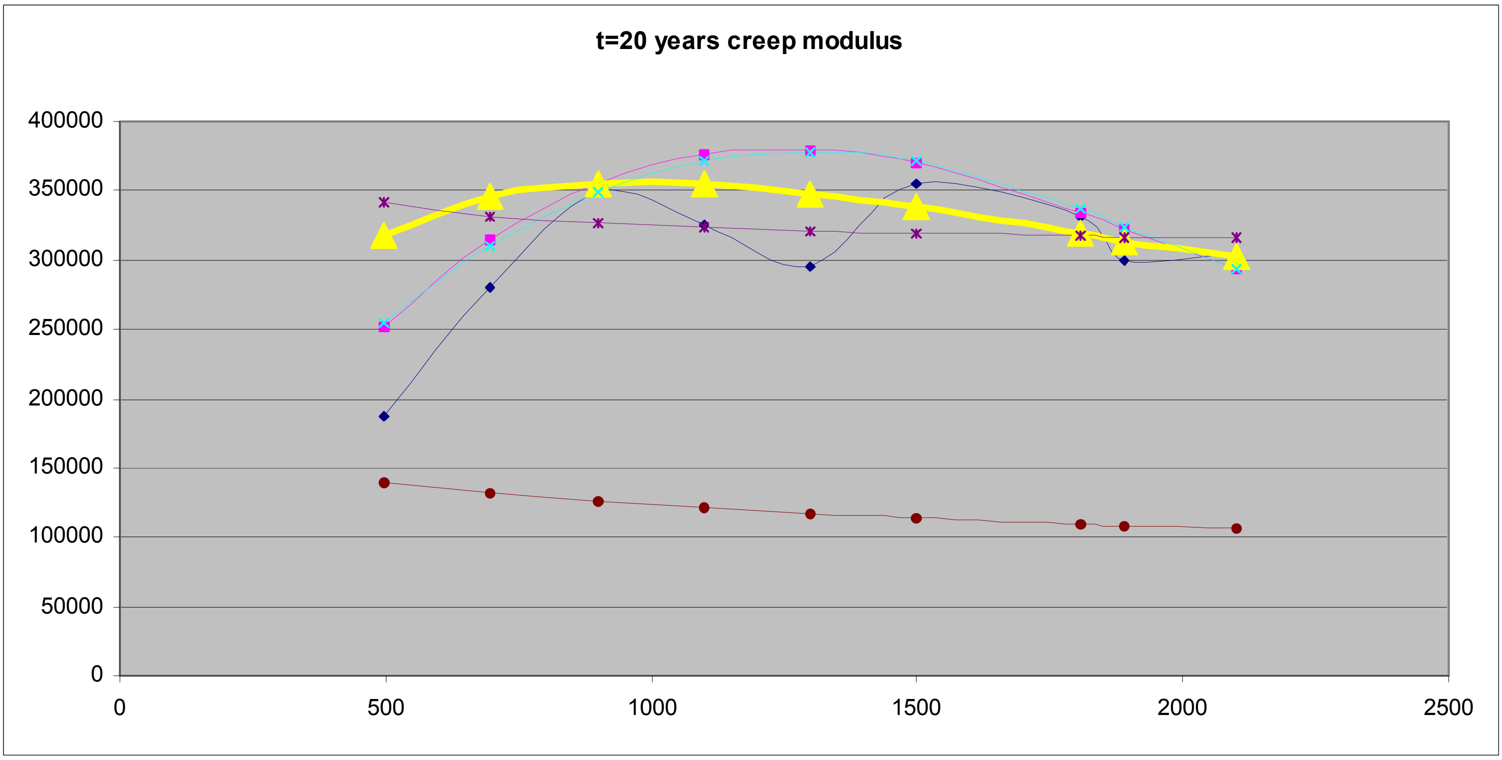
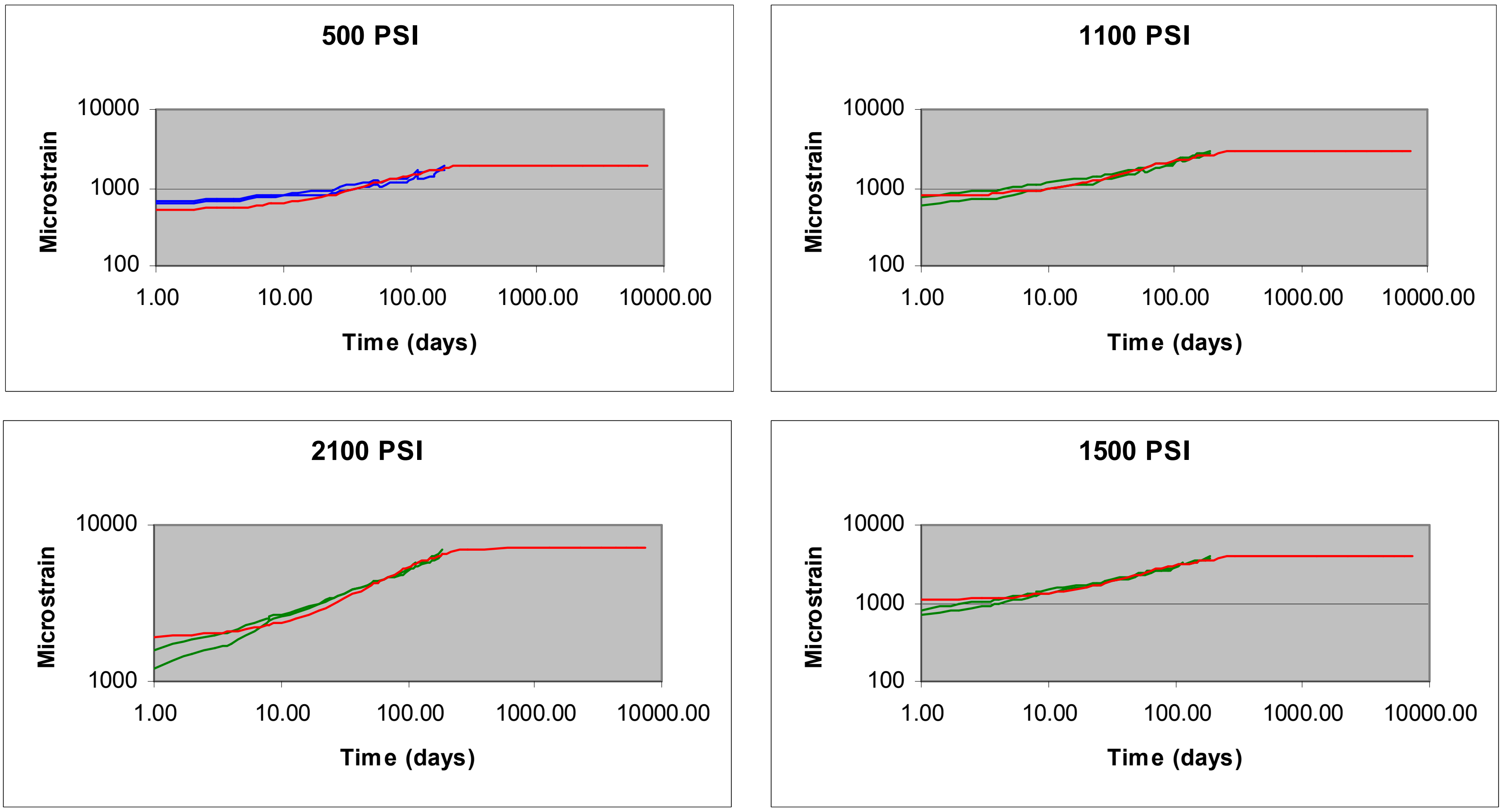


Figure 24: Stress vs. creep modulus at t = 20 years, predicted using a power law stress and exponential time fit function (bold yellow line)

$$\varepsilon = \left(\sigma^n + C\exp\left(\sigma/D\right)\right)\left(A + B\exp\left(-t/\tau\right)\right)$$



Figures 25-28: Creep data fit using an exponential time function and an exponential and power law stress function

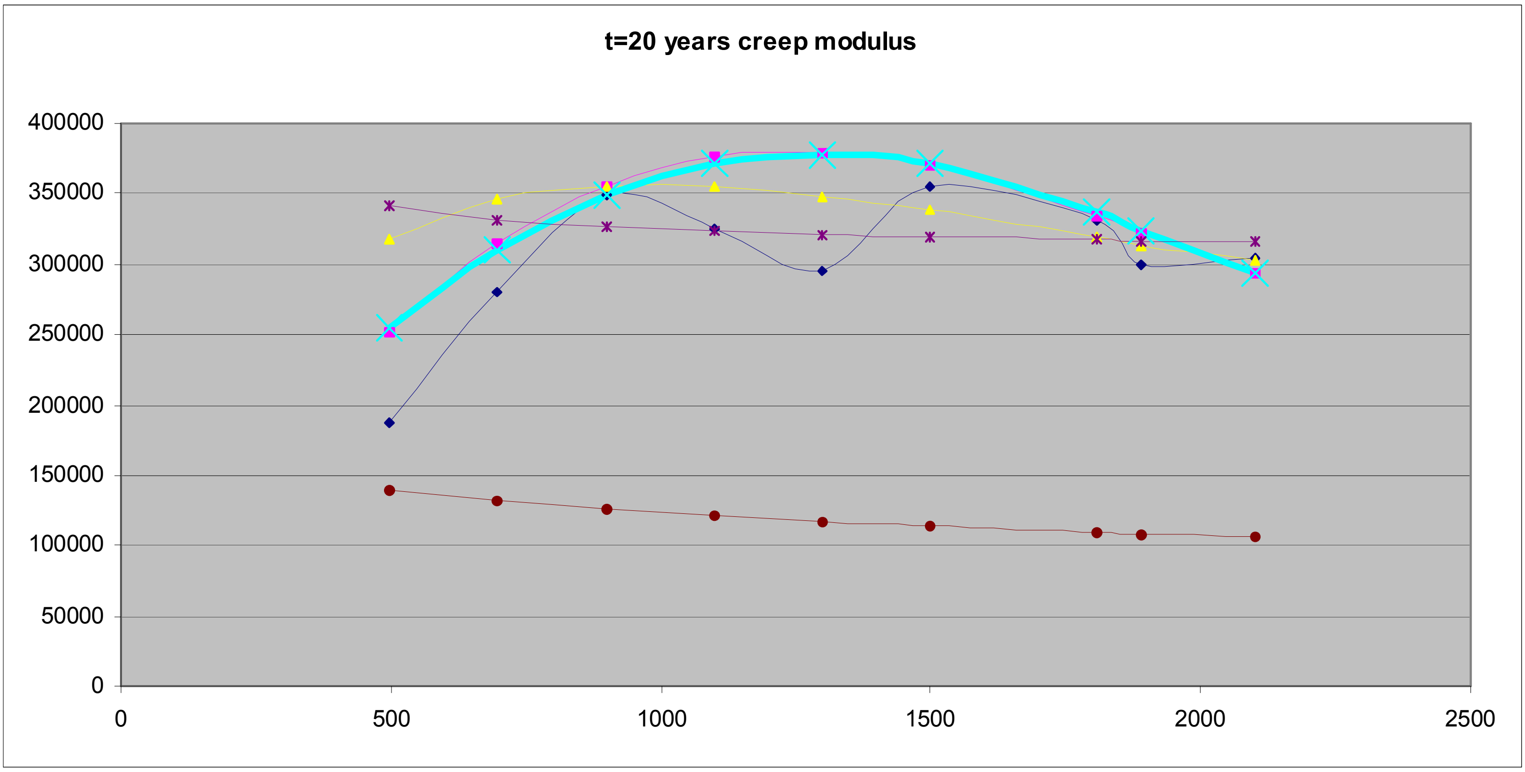
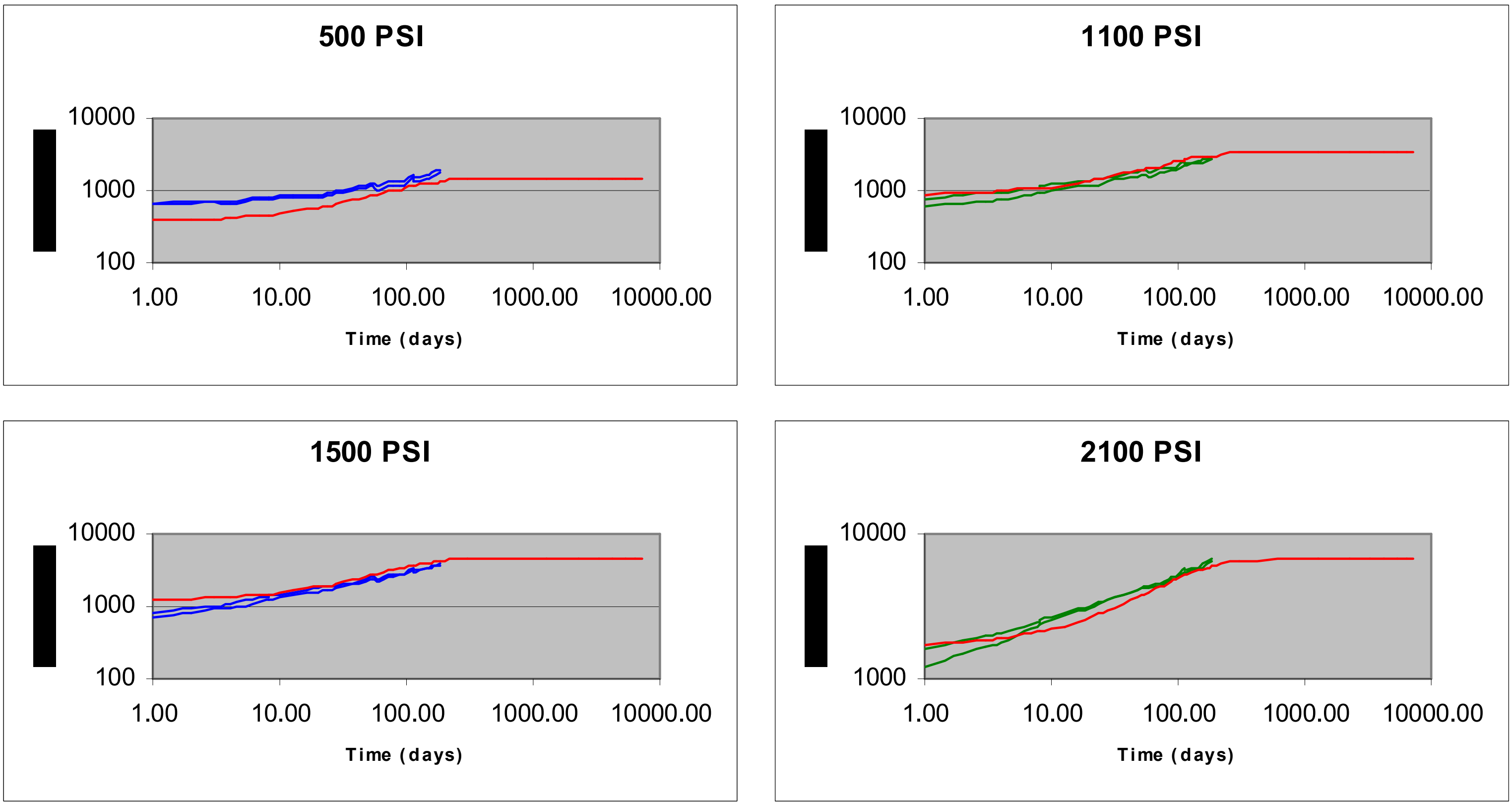


Figure 29: Stress vs. creep modulus at t = 20 years, predicted using a power law and exponential stress and exponential time fit function (bold light blue line)

$$\varepsilon = (D + C\sigma)\left(A + B\exp\left(-t/\tau\right)\right)$$



Figures 30-33: Creep data fit using an exponential time function and a linear stress function

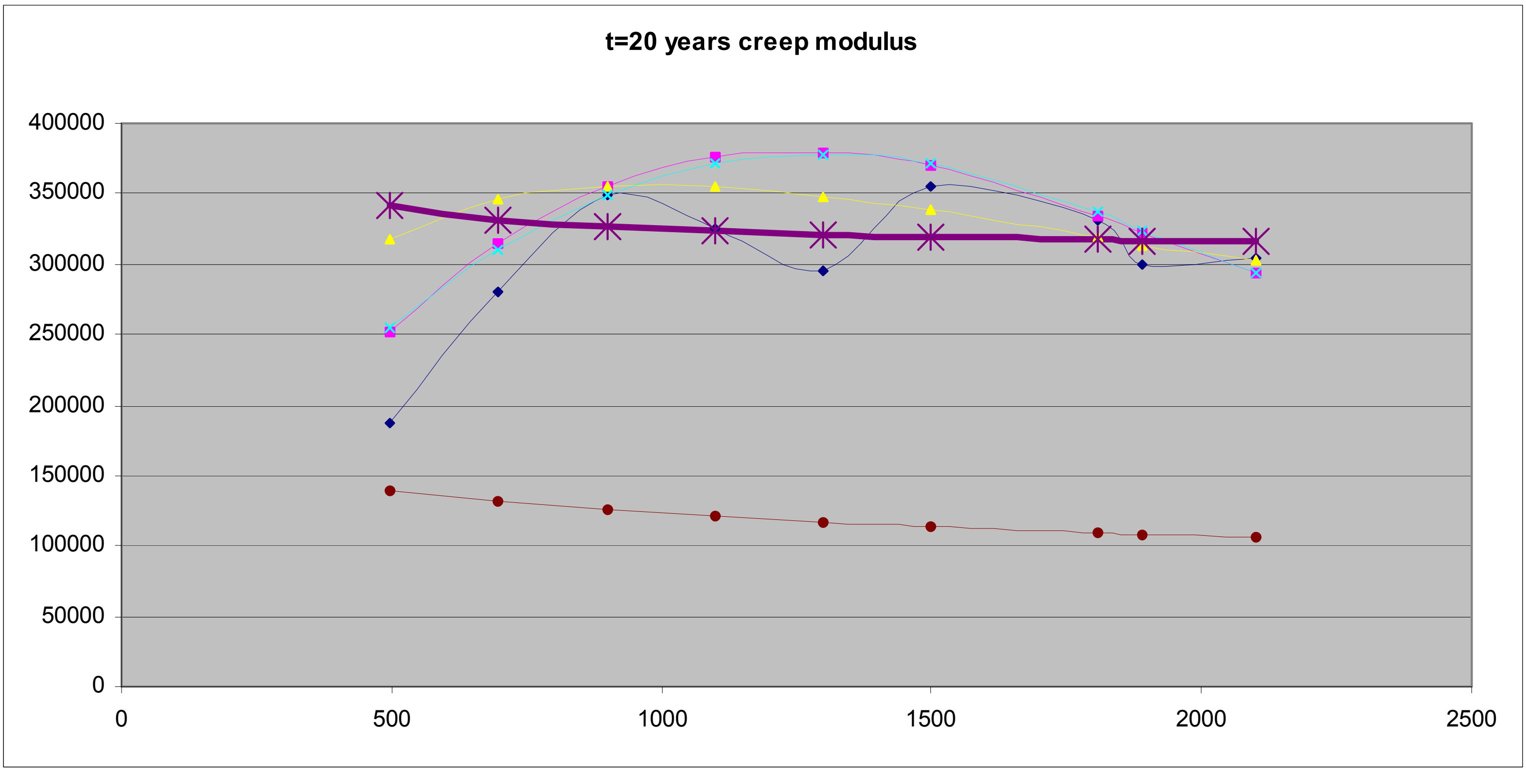
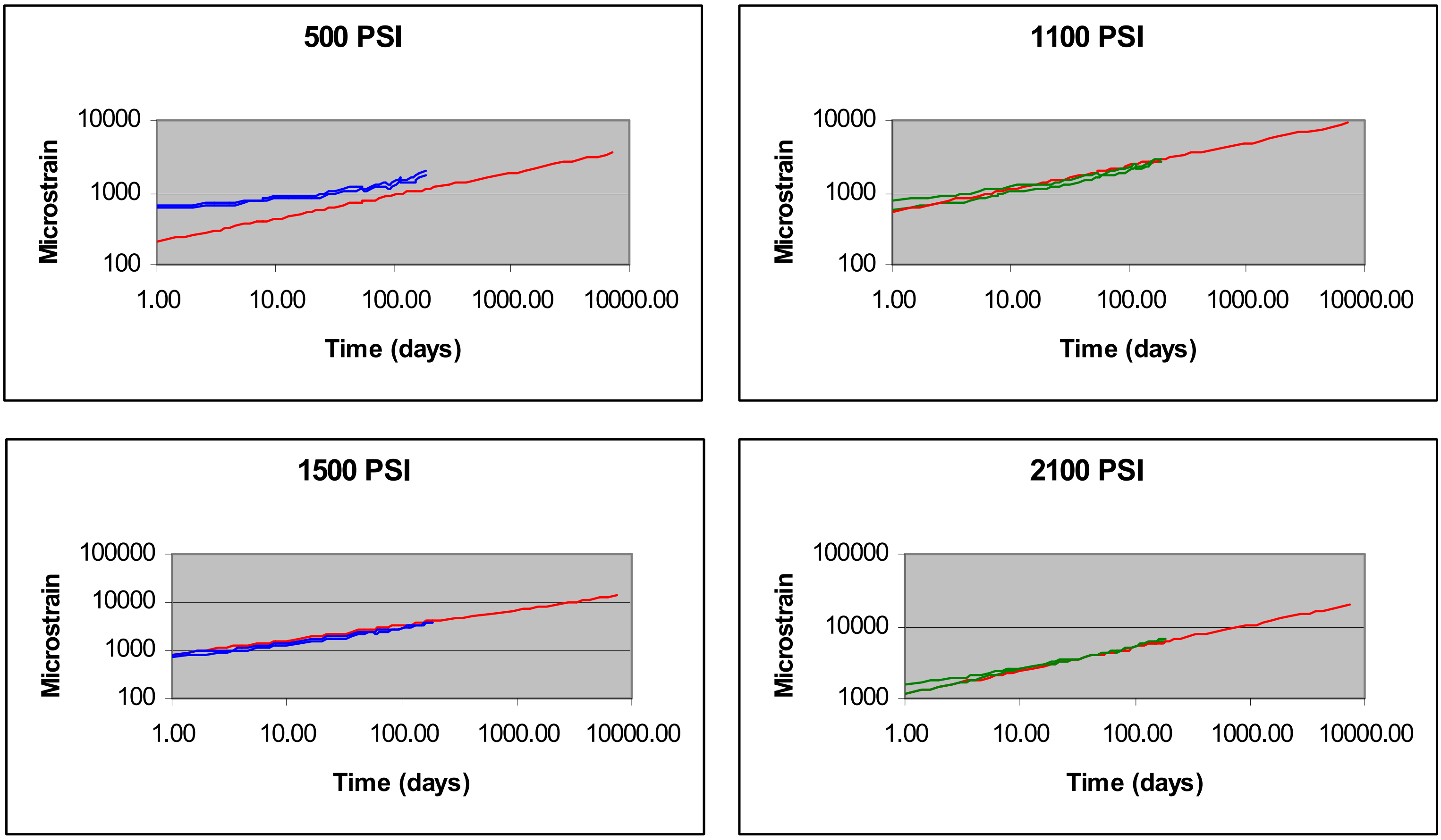


Figure 34: Stress vs. creep modulus at t = 20 years, predicted using a linear stress and exponential time fit function (bold purple line)

$$\varepsilon = A\sigma^m t^n$$



Figures 35-38: Creep data fit using an power law function

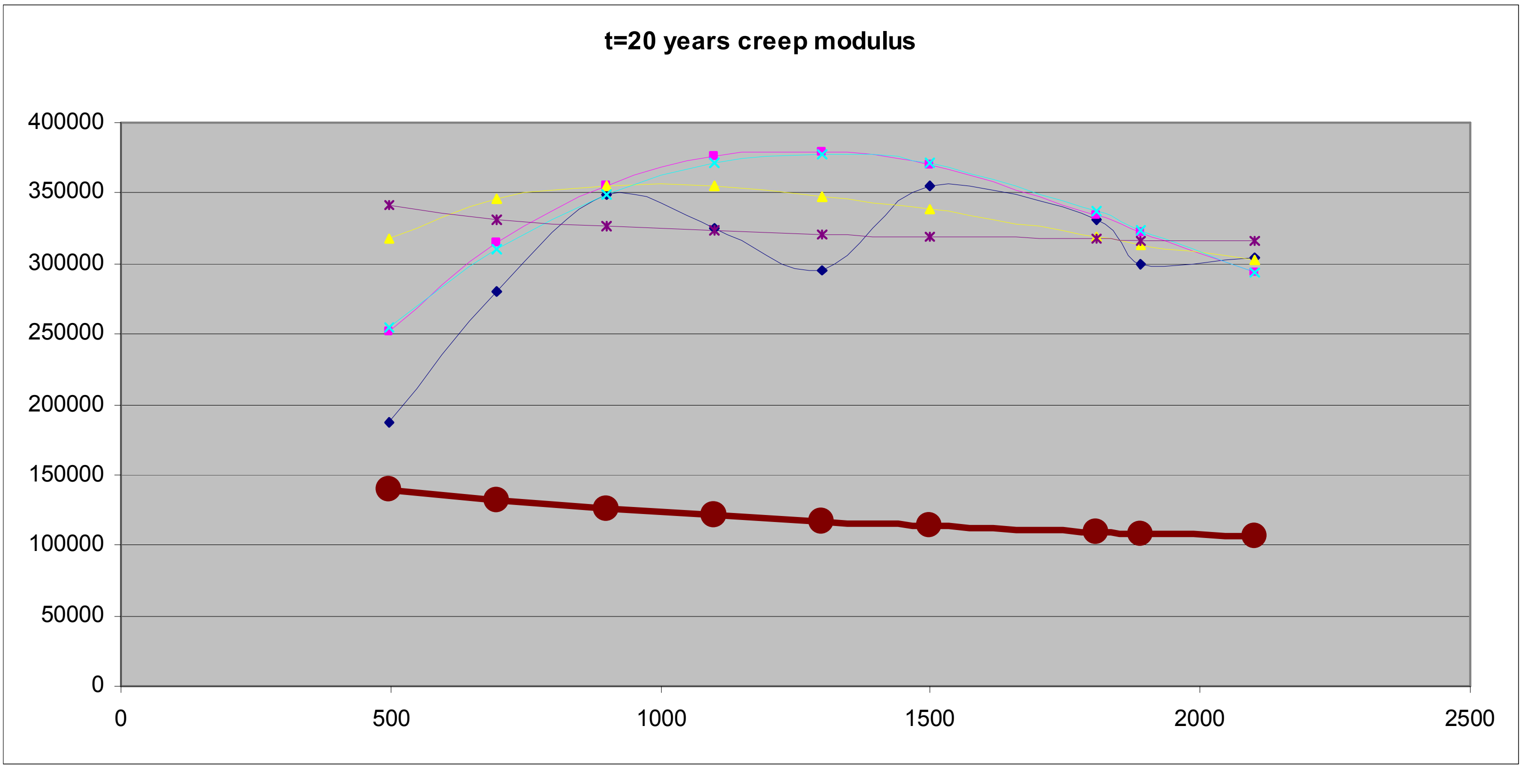
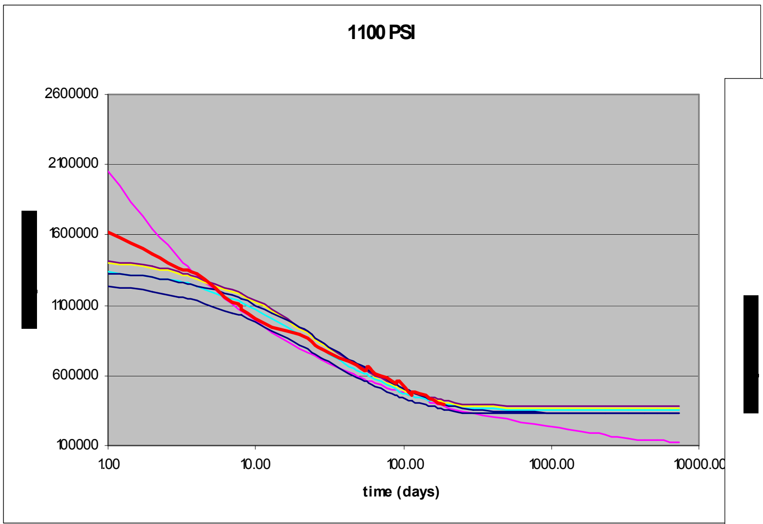


Figure 39: Stress vs. creep modulus at t = 20 years, predicted using a power law function (bold red line)

ANALYSIS/IMPLICATIONS

- For long times, linear viscoelastic model has a horizontal asymptote- this is not consistent with the expected shape of the strain curve, and it is not realistic to think that the strain rate will just go to zero and stay there.
- Power law fit continually increases at long times- since we expect to see a positive strain rate in both the secondary and tertiary creep stages, this may be more realistic
- The power law fit predicts well for high stress, and underpredicts for low stress
- The secondary creep stage has a constant strain rate, which means strain increases linearly.
- The power law fit increases proportionally to $t^{1/3}$
- If the creep reaches the secondary creep stage before 20 years, the strain will increase even faster than with the power law (this compares $t^{1/3}$ to t)

CREEP MODULUS VS. TIME



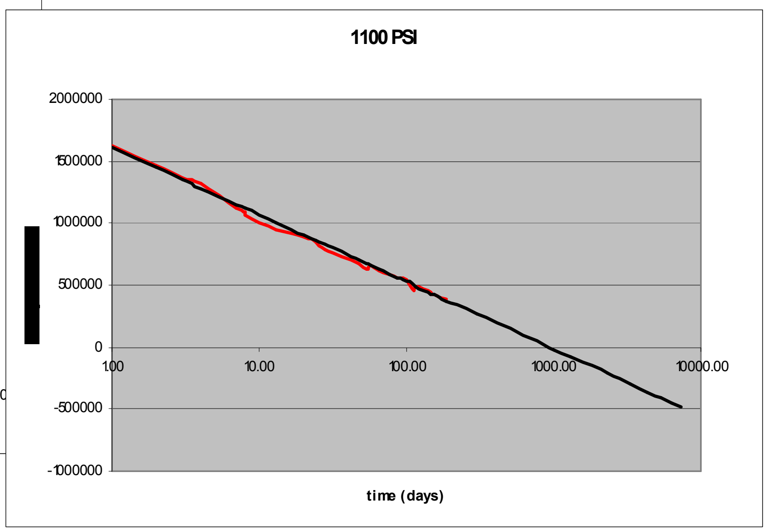


Figure 40: Creep modulus vs. time curves, extrapolated to 20 years, for all fits discussed earlier. The pink curve is the power law fit, and the bold red curve is the data. The rest of the curves are fits with exponential time dependence functions. (Note: x-axis is log scale, y-axis is linear scale)

Figure 41: Creep modulus vs. time curves, extrapolated to 20 years. The red curve is the data, and the black curve is a log fit function for the data curve. (Note: x-axis is log scale, y-axis is linear scale)

- Since I have done very little analysis on the creep modulus vs. time curves, this fit (figure 41) is included just to demonstrate the lack of predictive power that our current data set provides.
- It is clear from figure 40 that none of the fits gives the same curve as the data when comparing creep modulus to time.
- If the fit in figure 41 is accurate, the strain in the PVC would be infinite around 930 days, clearly indicating that the detector would not survive even 3 years
- Since I have no literature or data to support an assertion about fitting a creep modulus vs. time curve, I'm not trying to make a presumptuous assertion, however I felt that it was an important result to share.

CONCLUSIONS
- Linear viscoelastic theory only roughly fits the data for creep in PVC
- The data we have collected represents only the primary creep stage, which gives limited predictive power for later times
- There is no indication in the data that the secondary, or even tertiary, creep stages would not be reached in 20 years
- Based on this data, I think that an upper bound of 100,000-150,000 for the creep modulus at 20 years would be a conservative estimate
- In order to make a reasonable prediction of the strain on the PVC after 20 years, more data needs to be taken that includes the secondary and tertiary creep stages

NEXT STEPS
- Temperature dependent creep tests
  - This will hopefully give us data on the secondary and tertiary creep stages
  - The samples and test stands are ready for these tests to begin
- Determine an acceptable range of creep moduli for NO*ν*A
  - This should elucidate the implications of the creep analysis on the detector